6-2 Properties of Parallelograms

Common Core State Standards

G-CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other . . . **Also G-SRT.B.5**

MP 1, MP 3, MP 4

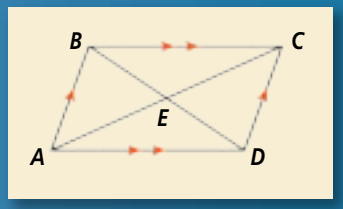
Objectives To use relationships among sides and angles of parallelograms
To use relationships among diagonals of parallelograms

Don't settle for your answer too soon! Be sure to find all pairs of congruent triangles.

SOLVE IT!

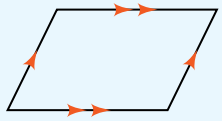
Getting Ready!

Use the information given in the diagram. Which triangles are congruent? How do you know?



MATHEMATICAL PRACTICES

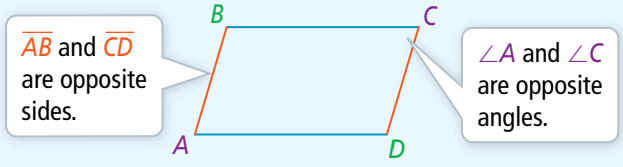
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In the Solve It, you made some conjectures about the characteristics of a parallelogram. In this lesson, you will verify whether your conjectures are correct.



Lesson Vocabulary
- parallelogram
- opposite sides
- opposite angles
- consecutive angles

Essential Understanding Parallelograms have special properties regarding their sides, angles, and diagonals.

In a quadrilateral, **opposite sides** do not share a vertex and **opposite angles** do not share a side.



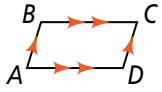
You can abbreviate *parallelogram* with the symbol $\square$ and *parallelograms* with the symbol $\square$s. You can use what you know about parallel lines and transversals to prove some theorems about parallelograms.

take note

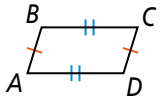
Theorem 6-3

Theorem
If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If . . .
$ABCD$ is a $\square$



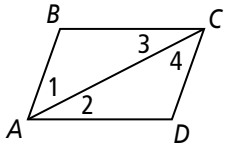
Then . . .
$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Proof Proof of Theorem 6-3

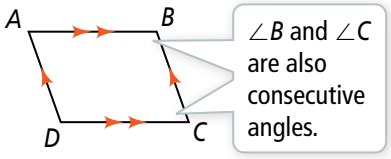
Given: $\square ABCD$

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Statements	Reasons
1) $ABCD$ is a parallelogram.	1) Given
2) $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$	2) Definition of parallelogram
3) $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 2$	3) If lines are $\parallel$, then alt. int. $\measuredangle$ are $\cong$.
4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Property of $\cong$
5) $\triangle ABC \cong \triangle CDA$	5) ASA
6) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$	6) Corresp. parts of $\cong$ $\triangle$ are $\cong$.

Angles of a polygon that share a side are **consecutive angles.** In the diagram, $\angle A$ and $\angle B$ are consecutive angles because they share side $\overline{AB}$.



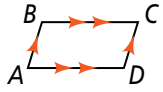
The theorem below uses the fact that consecutive angles of a parallelogram are same-side interior angles of parallel lines.

take note

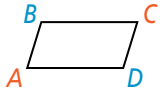
Theorem 6-4

Theorem
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If . . .
$ABCD$ is a $\square$



Then . . .



$m\angle A + m\angle B = 180$
$m\angle B + m\angle C = 180$
$m\angle C + m\angle D = 180$
$m\angle D + m\angle A = 180$

You will prove Theorem 6-4 in Exercise 32.

Problem 1 Using Consecutive Angles

Plan

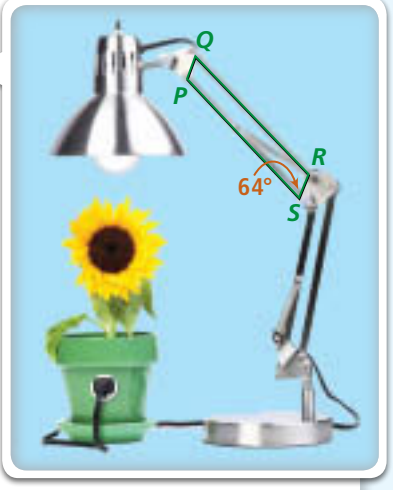
What information from the diagram helps you get started?
From the diagram, you know $m\angle PSR$ and that $\angle P$ and $\angle PSR$ are consecutive angles. So, you can *write an equation* and solve for $m\angle P$.

Multiple Choice What is $m\angle P$ in $\square PQRS$?

(A) 26 (C) 116
(B) 64 (D) 126

$m\angle P + m\angle S = 180$	Consecutive angles of a $\square$ are suppl.
$m\angle P + 64 = 180$	Substitute.
$m\angle P = 116$	Subtract 64 from each side.

The correct answer is C.



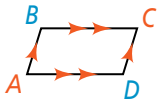
Got It? **1.** Suppose you adjust the lamp so that $m\angle S = 86$. What is $m\angle R$ in $\square PQRS$?

take note

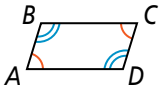
Theorem 6-5

Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If . . .
$ABCD$ is a $\square$.



Then . . .
$\angle A \cong \angle C$ and $\angle B \cong \angle D$



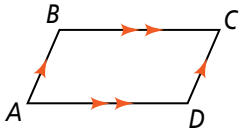
A proof of Theorem 6-5 in Problem 2 uses the consecutive angles of a parallelogram and the fact that supplements of the same angle are congruent.

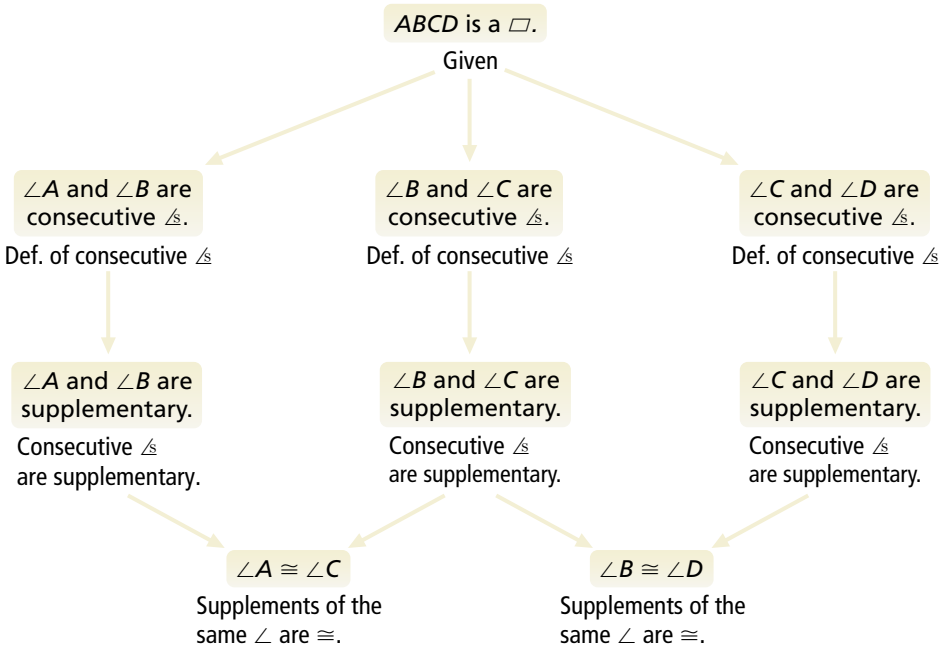
Proof

Problem 2 Using Properties of Parallelograms in a Proof

Given: $\square ABCD$

Prove: $\angle A \cong \angle C$ and $\angle B \cong \angle D$





Think

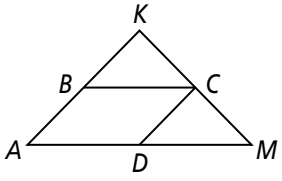
Why is a flow proof useful here?
A flow proof allows you to see how the pairing of two statements leads to a conclusion.

Got It?

2. Use the diagram at the right.

Given: $\square ABCD$, $\overline{AK} \cong \overline{MK}$

Prove: $\angle BCD \cong \angle CMD$



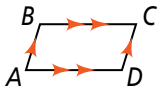
The diagonals of parallelograms have a special property.

take note

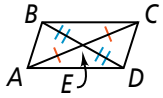
Theorem 6-6

Theorem
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If . . .
$ABCD$ is a $\square$



Then . . .
$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

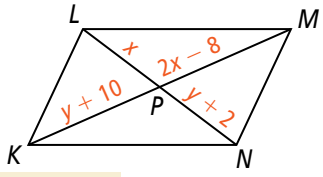


You will prove Theorem 6-6 in Exercise 13.

You can use Theorem 6-6 to find unknown lengths in parallelograms.

Problem 3 Using Algebra to Find Lengths

Solve a system of linear equations to find the values of x and y in $\square KLMN$. What are KM and LN?



Think

The diagonals of a parallelogram bisect each other.

$\overline{KP} \cong \overline{MP}$
$\overline{LP} \cong \overline{NP}$

Set up a system of linear equations by substituting the algebraic expressions for each segment length.

① $y + 10 = 2x - 8$
② $x = y + 2$

Substitute $(y + 2)$ for x in equation ①. Then solve for y.

$y + 10 = 2(y + 2) - 8$
$y + 10 = 2y + 4 - 8$
$y + 10 = 2y - 4$
$10 = y - 4$
$14 = y$

Substitute 14 for y in equation ②. Then solve for x.

$x = 14 + 2$
$= 16$

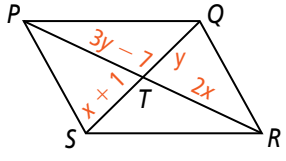
Use the values of x and y to find KM and LN.

$KM = 2(KP)$
$= 2(y + 10)$
$= 2(14 + 10)$
$= 48$

$LN = 2(LP)$
$= 2(x)$
$= 2(16)$
$= 32$

Got It? **3. a.** Find the values of x and y in $\square PQRS$ at the right. What are PR and SQ?

b. Reasoning In Problem 3, does it matter which variable you solve for first? Explain.



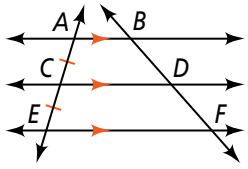
You will use parallelograms to prove the following theorem.

take note

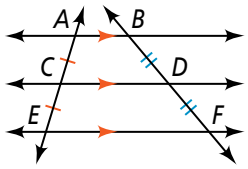
Theorem 6-7

Theorem
If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

If . . .
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$



Then . . .
$\overline{BD} \cong \overline{DF}$



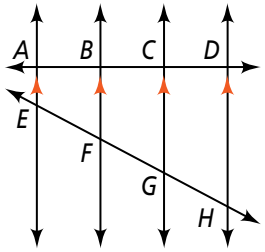
You will prove Theorem 6-7 in Exercise 43.

Problem 4 Using Parallel Lines and Transversals

Plan

What information do you need?
You know the length of $\overline{EF}$. To find EH, you need the lengths of $\overline{FG}$ and $\overline{GH}$.

In the figure, $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{DH}$, $AB = BC = CD = 2$, and $EF = 2.25$. What is EH?



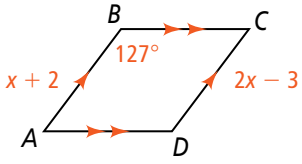
$EF = FG = GH$	Since $\parallel$ lines divide $\overline{AD}$ into equal parts, they also divide $\overline{EH}$ into equal parts.
$EH = EF + FG + GH$	Segment Addition Postulate
$EH = 2.25 + 2.25 + 2.25 = 6.75$	Substitute.

Got It? **4.** Use the figure in Problem 4. If $EF = FG = GH = 6$ and $AD = 15$, what is CD?

Lesson Check

Do you know HOW?

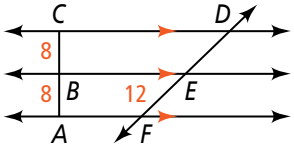
Use the diagram of $\square ABCD$ to find each value.



1. $m\angle A$ **2.** $m\angle D$

3. x **4.** AB

5. What are ED and FD in the figure at the right?

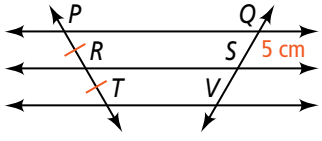


Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. Reasoning If you know one angle measure of a parallelogram, how do you find the other three angle measures? Explain.

7. Compare and Contrast What is the difference between a quadrilateral and a parallelogram?

8. Error Analysis Your classmate says that $QV = 10$. Explain why the statement may not be correct.



Practice and Problem-Solving Exercises

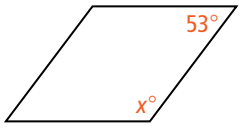
MATHEMATICAL PRACTICES

A Practice

Algebra Find the value of x in each parallelogram.

See Problem 1.

9.



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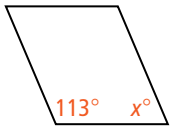

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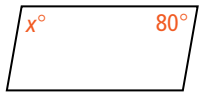
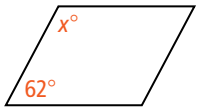


12.

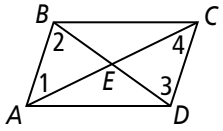


13. Developing Proof Complete this two-column proof of Theorem 6-6.

See Problem 2.

Given: $\square ABCD$

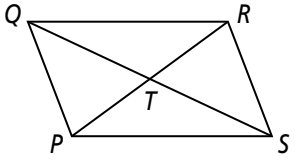
Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.



Statements	Reasons
1) $ABCD$ is a parallelogram.	1) Given
2) $\overline{AB} \parallel \overline{DC}$	2) a. ?
3) $\angle 1 \cong \angle 4; \angle 2 \cong \angle 3$	3) b. ?
4) $\overline{AB} \cong \overline{DC}$	4) c. ?
5) d. ?	5) ASA
6) $\overline{AE} \cong \overline{CE}; \overline{BE} \cong \overline{DE}$	6) e. ?
7) f. ?	7) Definition of bisector

Algebra Find the values of x and y in $\square PQRS$.

See Problem 3.



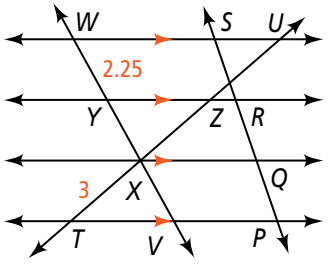
14. $PT = 2x$, $TR = y + 4$, $QT = x + 2$, $TS = y$

15. $PT = x + 2$, $TR = y$, $QT = 2x$, $TS = y + 3$

16. $PT = y$, $TR = x + 3$, $QT = 2y$, $TS = 3x - 1$

In the figure, $PQ = QR = RS$. Find each length.

See Problem 4.



17. ZU

18. XZ

19. TU

20. XV

21. YX

22. YV

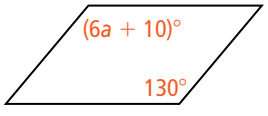
23. WX

24. WV

B Apply

Algebra Find the value(s) of the variable(s) in each parallelogram.

25.



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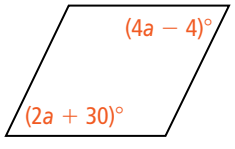

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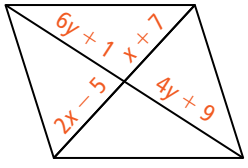
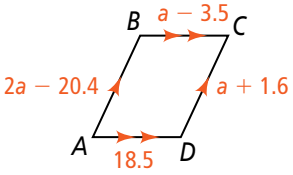


28. Think About a Plan What are the values of x and y in the parallelogram?

- How are the angles related?
- Which variable should you solve for first?

Algebra **Find the value of a. Then find each side length or angle measure.**

29.



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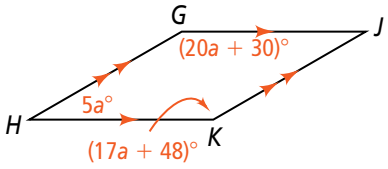
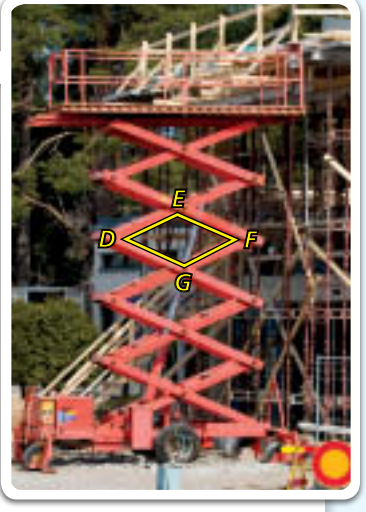


31. Studio Lighting A pantograph is an expandable device shown at the right. Pantographs are used in the television industry in positioning lighting and other equipment. In the photo, points D, E, F, and G are the vertices of a parallelogram. $\square DEFG$ is one of many parallelograms that change shape as the pantograph extends and retracts.

a. If $DE = 2.5$ ft, what is FG? **b.** If $m\angle E = 129$, what is $m\angle G$?

c. What happens to $m\angle D$ as $m\angle E$ increases or decreases? Explain.

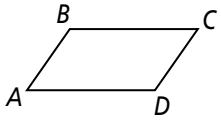


32. Prove Theorem 6-4.

Proof

Given: $\square ABCD$

Prove: $\angle A$ is supplementary to $\angle B$.
$\angle A$ is supplementary to $\angle D$.



Use the diagram at the right for each proof.

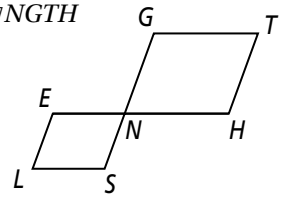
Proof

33. Given: $\square LENS$ and $\square NGTH$

Prove: $\angle L \cong \angle T$

34. Given: $\square LENS$ and $\square NGTH$

Prove: $\overline{LS} \parallel \overline{GT}$



35. Given: $\square LENS$ and $\square NGTH$

Prove: $\angle E$ is supplementary to $\angle T$.

Use the diagram at the right for each proof.

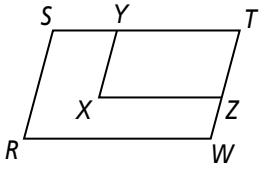
Proof

36. Given: $\square RSTW$ and $\square XYTZ$

Prove: $\angle R \cong \angle X$

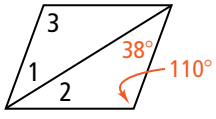
37. Given: $\square RSTW$ and $\square XYTZ$

Prove: $\overline{XY} \parallel \overline{RS}$

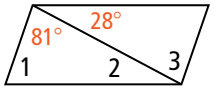


Find the measures of the numbered angles for each parallelogram.

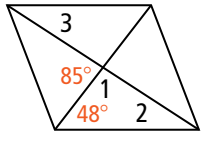
38.



39.



40.



41. Algebra The perimeter of $\square ABCD$ is 92 cm. AD is 7 cm more than twice AB. Find the lengths of all four sides of $\square ABCD$.

Challenge

42. Writing Is there an SSSS congruence theorem for parallelograms? Explain.

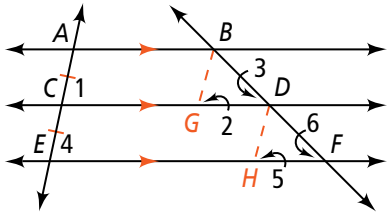
43. Prove Theorem 6-7. Use the diagram at the right.

Proof

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, $\overline{AC} \cong \overline{CE}$

Prove: $\overline{BD} \cong \overline{DF}$

(*Hint:* Draw lines through B and D parallel to $\overleftrightarrow{AE}$ and intersecting $\overleftrightarrow{CD}$ at G and $\overleftrightarrow{EF}$ at H.)

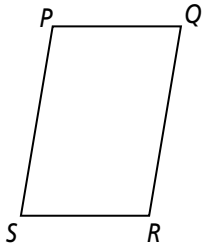


44. Measurement Explain how to separate a blank card into three strips that are the same height by using lined paper, a straightedge, and Theorem 6-7.

Standardized Test Prep

SAT/ACT

45. $PQRS$ is a parallelogram with $m\angle Q = 4x$ and $m\angle R = x + 10$. Which statement explains why you can use the equation $4x + (x + 10) = 180$ to solve for x?



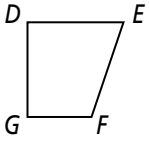
(A) The measures of the interior angles of a quadrilateral have a sum of 360.

(B) Opposite sides of a parallelogram are congruent.

(C) Opposite angles of a parallelogram are congruent.

(D) Consecutive angles of a parallelogram are supplementary.

46. In the figure of $DEFG$ at the right, $\overline{DE} \parallel \overline{GF}$. Which statement must be true?



(F) $m\angle D + m\angle E = 180$

(G) $m\angle D + m\angle G = 180$

(H) $\overline{DE} \cong \overline{GF}$

(I) $\overline{DG} \cong \overline{EF}$

47. An obtuse triangle has side lengths of 5 cm, 9 cm, and 12 cm. What is the length of the side opposite the obtuse angle?

(A) 5 cm (B) 9 cm (C) 12 cm (D) not enough information

Short Response

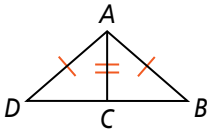
48. Find the measure of one exterior angle of a regular hexagon. Explain your method.

Mixed Review

Find the sum of the measures of the interior angles of each polygon. See Lesson 6-1.

49. decagon **50.** 16-gon **51.** 25-gon **52.** 40-gon

53. What additional information do you need to prove $\triangle ADC \cong \triangle ABC$ by the HL Theorem? See Lesson 4-6.



Get Ready! **To prepare for Lesson 6-3, do Exercise 54.**

54. Two consecutive angles in a parallelogram have measures $x + 5$ and $4x - 10$. Find the measure of the smaller angle. See Lesson 6-2.